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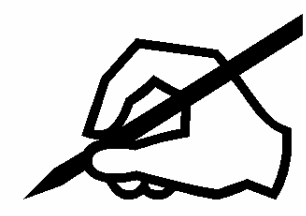

LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT Five:
Completing the Square

Lesson
9

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1 and 2

(Some helpful exercises and page numbers are given throughout the lesson, e.g. (Ex 13l page 790)

INTRODUCTION

In this lesson students will be exposed to the skills involved in completing the square. Some of the skills developed in lesson eight, for example factorization by grouping and manipulation of signed numbers, will also be applied here.

OBJECTIVES

At the end of this lesson you will be able to:

a) complete the square for quadratic equations

b) solve quadratic equations by completing the square.

5.7 Completing the square

When you complete the square you write the quadratic in the form $a(x+h)^2+k$, where $a(x+h)^2$ is a perfect square. The following example will be helpful.

Example:

Write the following quadratic expression in the form $a(x+h)^2+k$:

$3x^2+4x+5$.

Solution:

Enclose the variable terms in brackets.

$(3x^2+4x)+5$

'Factor out' the coefficient (the number in front) of x^2.

$3\left(x^2+\frac{4}{3}x\right)+5$

Get half of the coefficient of x i.e. $\frac{4}{3}\times\frac{1}{2}=\frac{2}{3}$.

Square this and add it to the variables in the bracket.

$3\left(x^2+\frac{4}{3}x+\left(\frac{2}{3}\right)^2\right)+5$

$\left(\frac{2}{3}\right)^2$ has to be subtracted 3 times from the 5 because it was added 3 times to the terms in the brackets (It was added three times because of the 3 outside the brackets).

$3\left(x^2+\frac{4}{3}x+\left(\frac{2}{3}\right)^2\right)+5-3\times\left(\frac{2}{3}\right)^2$

Split the $\frac{4}{3}x$ term using the fact that $\frac{2}{3}x+\frac{2}{3}x=\frac{4}{3}x$. Simplify the terms outside the brackets.

$$3\left(x^2+\frac{2}{3}x+\frac{2}{3}x+\left(\frac{2}{3}\right)^2\right)+5-3\times\frac{4}{9}$$

Factorize the terms inside the brackets by grouping. Continue to simplify the terms outside the brackets.

$$3\left[x\left(x+\frac{2}{3}\right)+\frac{2}{3\left(x+\frac{2}{3}\right)}\right]+5-\frac{4}{3}$$

'Factor out' $\left(x+\frac{2}{3}\right)$ from the square brackets. Continue to simplify the terms outside the brackets.

$$3\left(x+\frac{2}{3}\right)\left[x+\frac{2}{3}\right]+5-1\frac{1}{3}$$

$$3\left(x+\frac{2}{3}\right)^2+3\frac{2}{3}$$

Note: $a=3, h=\frac{2}{3}\ and\ k=3\frac{2}{3}$

Write the following quadratic expression in the form $a(x+h)^2+k$ (Ex 13l page 790):

$7x^2-14x-9$.

Completing the square where the coefficient of x^2 is negative.

Write the following quadratic expression in the form $k + a(x + h)^2$:

$-10x^2 + 5x - 8$.

Solution:

$(-10x^2 + 5x) - 8$

'Factor out' -10 , note the change in sign in the brackets.

$-10\left(x^2 - \frac{5}{10}x\right) - 8$

Note $\frac{5}{10} \equiv \frac{1}{2}$ so we can write:

$-10\left(x^2 - \frac{1}{2}x\right) - 8$

Half the coefficient of x is $-\frac{1}{4}$, if we square this term and add as before we get:

$-10\left(x^2 - \frac{1}{2}x + \left(\frac{1}{4}\right)^2\right) - 8 + 10 \times \left(\frac{1}{4}\right)^2$

Because of the -10 outside of the brackets we actually subtracted $\left(\frac{1}{4}\right)^2$ ten times from the terms in the brackets therefore to keep the expression unchanged we must added it ten times to the term outside the brackets.

Split the middle term using the fact that $-\frac{1}{4}x - \frac{1}{4}x = -\frac{1}{2}x$. Continue to simplify the terms outside the brackets.

$$-10\left(x^2-\frac{1}{4}x-\frac{1}{4}x+\left(\frac{1}{4}\right)^2\right)-8+10\times\frac{1}{16}$$

Factorize the terms inside the brackets by grouping. Continue to simplify the terms outside the brackets.

$$-10\left[x\left(x-\frac{1}{4}\right)-\frac{1}{4}\left(x-\frac{1}{4}\right)\right]-8+\frac{5}{8}$$

'Factor out' $\left(x-\frac{1}{4}\right)$ from the square brackets. Continue to simplify the terms outside the brackets.

$$-10\left(x-\frac{1}{4}\right)\left[x-\frac{1}{4}\right]-7\frac{3}{8}$$

$$-10\left(x-\frac{1}{4}\right)^2-7\frac{3}{8}$$

This can be written:

$$-7\frac{3}{8}-10\left(x-\frac{1}{4}\right)^2$$

$$a=-10, \qquad h=-\frac{1}{4}\ and\ k=-7\frac{3}{8}$$

ACTIVITY 2

Write the following quadratic expression in the form $k + a(x + h)^2$ (Ex 13l page 790):

$-8x^2 - 4x + 5$.

Solving quadratics by completing the square

Quadratic equations can be easily solved after completing the square.

Example:

Solve the following quadratic equation(Ex 13l page 790):

$3x^2 + 6x + 1 = 0$

By completing the square we have:

$3(x + 1)^2 - 2 = 0$

Now transpose for x .

$3(x + 1)^2 - 2 + 2 = 0 + 2$

$3(x + 1)^2 = 2$

$$\frac{3(x+1)^2}{3} = \frac{2}{3}$$

$$(x + 1)^2 = \frac{2}{3}$$

$$\sqrt{(x+1)^2} = \pm\sqrt{\frac{2}{3}}$$

$$x + 1 = \pm\sqrt{\frac{2}{3}}$$

$$x + 1 - 1 = \pm\sqrt{\frac{2}{3}} - 1$$

$$x = \pm\sqrt{\frac{2}{3}} - 1$$

It is a quadratic so normally we expect two solutions. The square root gives a $\pm$ this is how we get the two solutions. Therefore:

$$x = \sqrt{\frac{2}{3}} - 1 \qquad x = -\sqrt{\frac{2}{3}} - 1$$

$$x = -0.18 \qquad x = -1.82$$

Solve the following quadratic equation(Ex 13l page 790):

$-4x^2 - 5x + 2 = 0$

C X C question

1. Write the expression $9x^2 - 9x + 1$ in the form $a(x + b)^2 + c$, where $a, b \text{ and } c$ real numbers.

2. Solve the equation $4x^2 = 9x - 1$

giving your answers to two decimal points.

CONCLUSION

We have looked at completing the square and its application in solving quadratic equations. In the lesson that follows we will look at solving simultaneous equations and inequalities.